# On a Constant in the Theory of Trigonometric Series

By Robert F. Church

The note "A constant in the theory of trigonometric series" in the October 1964 issue of *Mathematics of Computation* provided us with a test for our recently constructed algorithms for the computation of roots of functions, and for numerical quadrature in the presence of singularities. The latter algorithm, utilizing the Gaussian 8-point quadrature formula applied to sub-intervals of variable length, involves a sufficiently small number of ordinates that computational labor and round-off error do not become problems. Use of these algorithms indicated the value $\alpha_0 = .3084438$, for the root of the equation $\int_0^{3\pi/2} u^{-\alpha} \cos u \, du = 0$, differing from the reported value, .30483, in the third place. To check this result, we made the transformation $u = x^4$ to weaken the character of the singularity at the origin, and obtained the following table by conventional numerical quadrature, confirming our result:

| $\alpha$ | $F(\alpha)$ |
|---|---|
| .308441 | $-.99\,(10^{-5})$ |
| .308442 | $-.63\,(10^{-5})$ |
| .308443 | $-.28\,(10^{-5})$ |
| .308444 | $.08\,(10^{-5})$ |
| .308445 | $.44\,(10^{-5})$ |
| .308446 | $.79\,(10^{-5})$. |

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In a recent note, Boas and Klema [1] considered

$$F(\alpha) = \int_0^{3\pi/2} u^{-\alpha} \cos u \, du, \qquad R(\alpha) < 1, \tag{1}$$

and gave some computations from which they concluded that a zero $\alpha_0$ of $F(\alpha)$ lies between 0.30483 and 0.30484. Since their tabulated values of $F(\alpha)$ in the vicinity of the root are given to 8D and there are eight such entries, it would seem, since $F(\alpha)$ is analytic for $R(\alpha) < 1$, that the zero could be given to more places by differencing and making use of ordinary inverse interpolation techniques. It is found

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